Main Ideas in Class Today

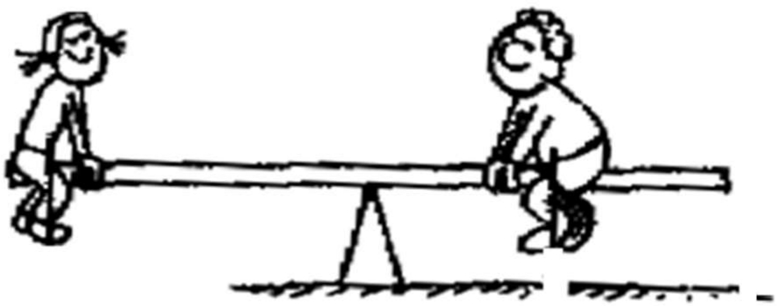
You should be able to:

- Compute center of gravity
- Understand why some things are more difficult to rotate
- Utilize the rotational equivalent to conservation of (angular) momentum
- Understand Moment of Inertia, Rotational Kinetic Energy and Angular Momentum

Extra Practice: 8.39, 8.43, 8.49, 8.53, 8.55, 8.57, 8.61, 8.63, 8.73, 8.85

Center of Gravity/Mass

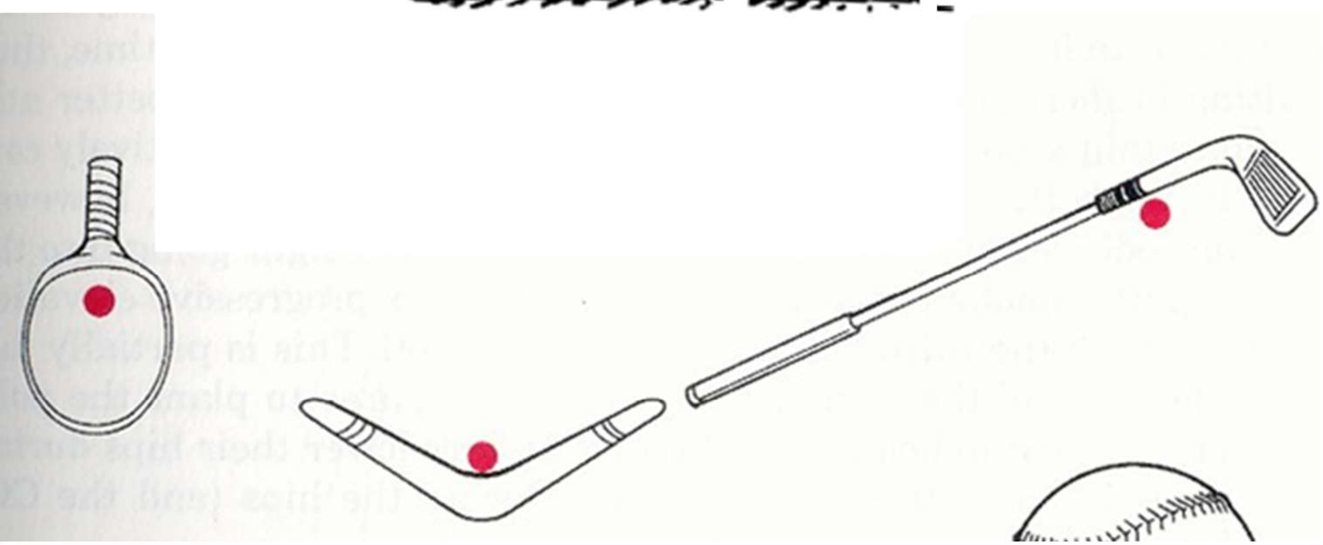
Find the center of gravity of the system of two blocks above if they are 50 m apart.



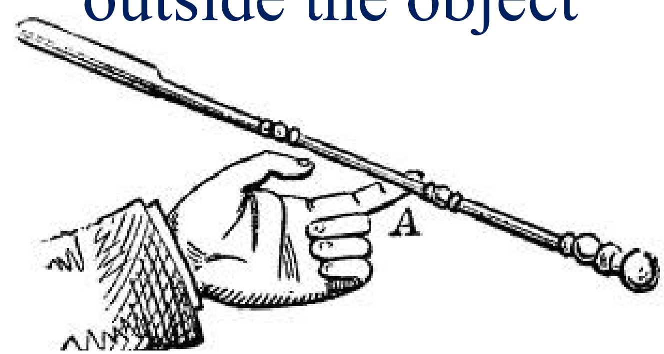
The center of gravity is the point around which a body's mass is equally distributed in all directions.

In a uniform object, at its mid point

Center of gravity can be outside the object



$$x_{cg} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\Sigma m_i x_i}{\Sigma m_i}$$



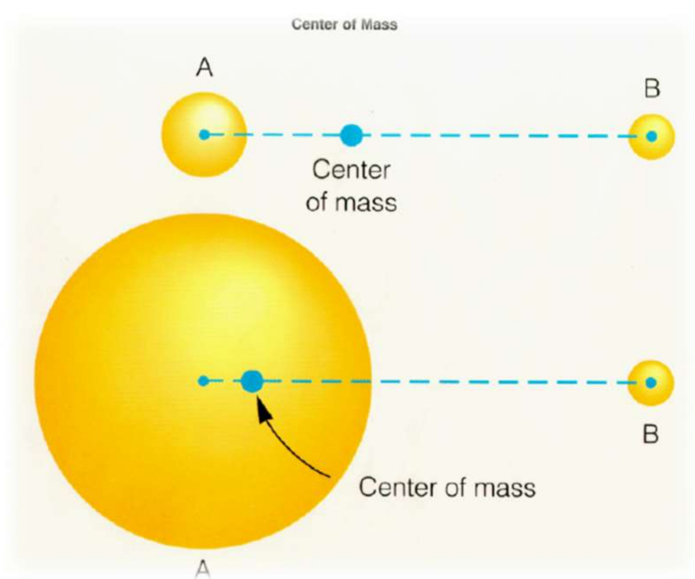
Center of Mass in Astrophysics

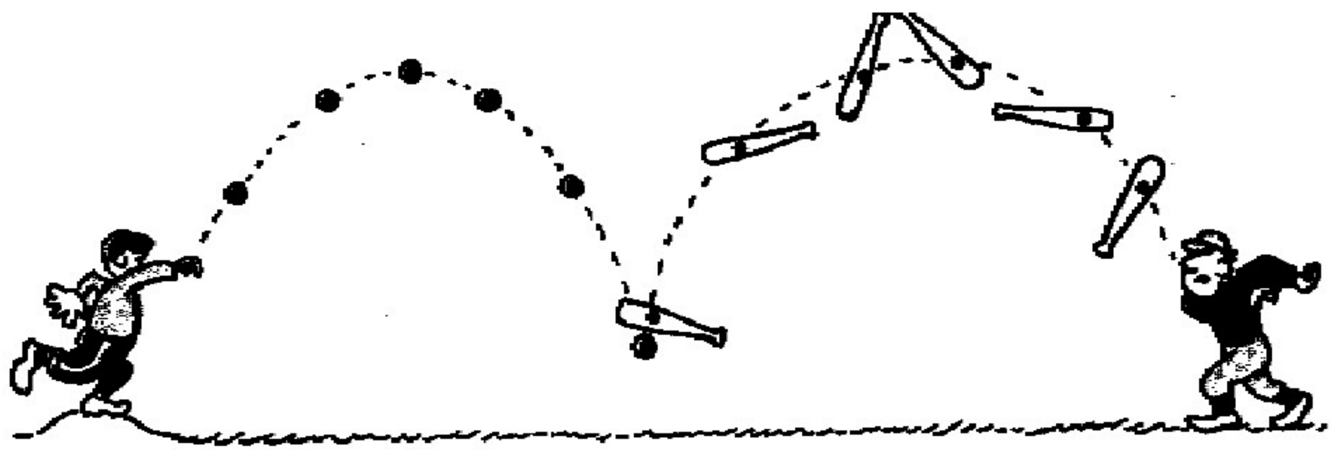
Even star/planet systems rotate about center of mass

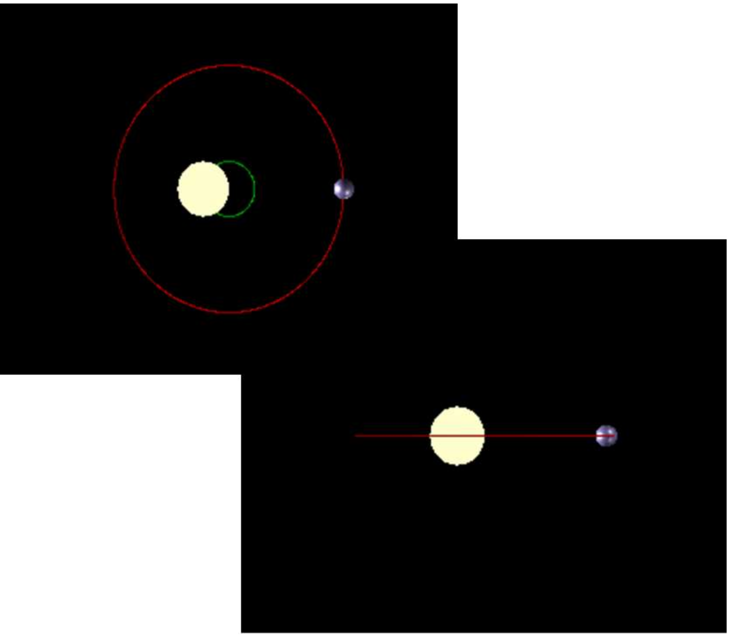
Center of mass of solar system is not in the center of the star, causing the star to wobble a little

Astronomers search for wobbling stars to find planets

As of October 2024, over 5600 extrasolar planets have been confirmed.



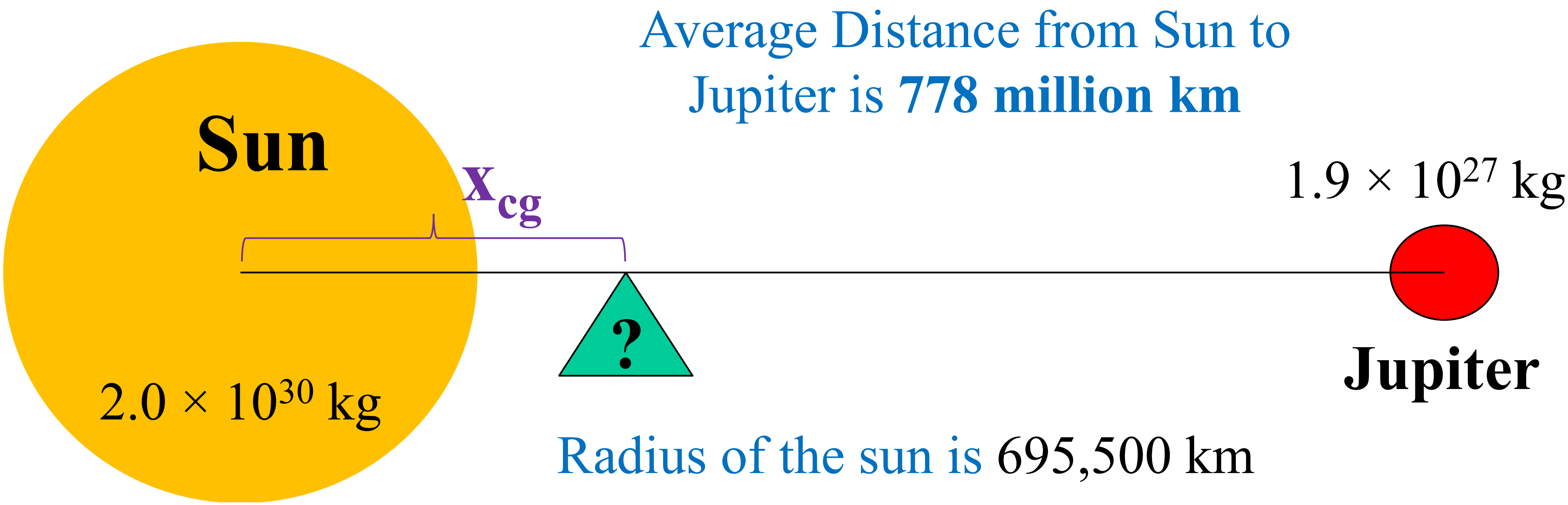




Center of Mass in Astrophysics

Star/planet systems (and other objects) rotate about center of mass

Imagine if our solar system only consisted of the Sun and Jupiter. Where would the center of mass be?



Everything has a rotational equivalent

What formulas do we still need to look at the equivalents of?
Ch. 4 Forces, Ch. 5 Energy, Ch. 6 Momentum

Linear	Rotational	
Δx	$\Delta\theta$	$\Delta x = r\Delta\theta$
$\bar{v} = \dfrac{\Delta x}{\Delta t}$	$\bar{\omega} = \dfrac{\Delta\theta}{\Delta t}$	$v = r\omega$
$\bar{a} = \dfrac{\Delta v}{\Delta t}$	$\bar{\alpha} = \dfrac{\Delta\omega}{\Delta t}$	$a = r\alpha$

Note the line means average!

For constant a:

$$v = v_o + at$$

$$\Delta x = v_o t + \tfrac{1}{2}at^2$$

$$v^2 = v_o^2 + 2a\Delta x$$

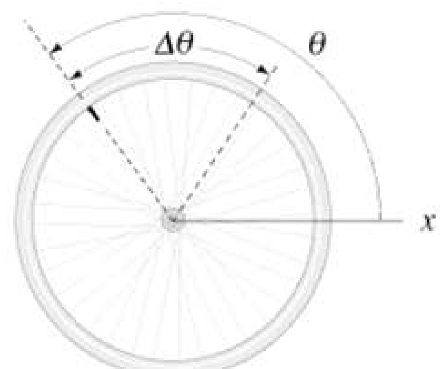
If the radius does not change

For constant α:

$$\omega = \omega_o + \alpha t$$

$$\Delta\theta = \omega_o t + \tfrac{1}{2}\alpha t^2$$

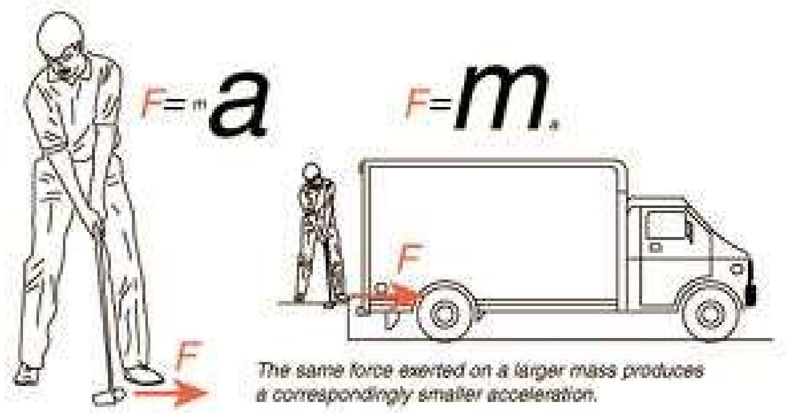
$$\omega^2 = \omega_o^2 + 2\alpha\Delta\theta$$



Moment of Inertia

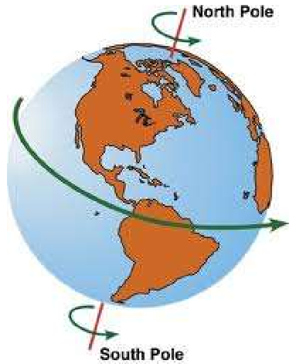
- An object's mass tells us how difficult it is to push by the formula:

$\Sigma F = ma$ (Newton's 2nd law)



- An object's **moment of inertia** (I) tells us how difficult it is to rotate by the formula $\Sigma\tau = I\alpha$

where α = angular acceleration



Rotational Inertia

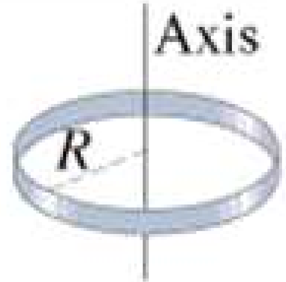
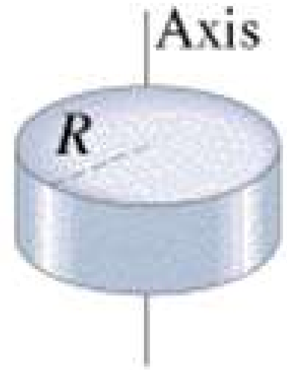
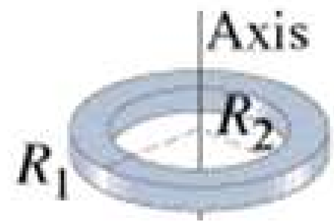
$\Sigma\tau = I\alpha$

The angular velocity of an object does not change ($\alpha=0$) unless acted on by a torque

Compare to Newton's 1st law: The velocity of an object does not change ($a=0$) unless acted on by a force $\Sigma F = ma$

Rotational Inertia depends not just on mass but also how the mass is distributed

Examples

	Object	Location of axis		Moment of inertia
A	Thin hoop of radius R	Through center		MR^2
B	Solid cylinder of radius R	Through center		$\frac{1}{2}MR^2$
C	Hollow cylinder of inner radius R_1 and outer radius R_2	Through center		$\frac{1}{2}M(R_1^2 + R_2^2)$ $R_1 \neq R_2$

If the mass and outer R is the same, which one of these 3 I's is smallest?

D. Two are the same

$I = \Sigma MR^2$

Q108

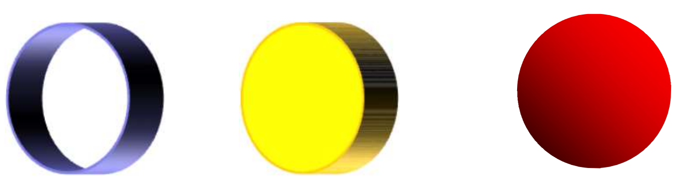
Do not memorize, will be given if needed

All shapes below have the same mass and same radius

In order to maximize the moment of inertia (I), the mass should be:

$I =$ MR^2 $\frac{1}{2} MR^2$ $\frac{2}{5} MR^2$

Hoop Disk Sphere



A. Concentrated at the edges

B. Evenly distributed

C. Concentrated at the center

D. Makes no difference

Q109

Race of the Geometrical Shapes

All shapes below have the same mass and same radius

Since all of these shapes have the **same mass**, they all have the same force that will act to accelerate them down the incline (torque).

Which shape will win the race?

A. Hoop

B. Disk

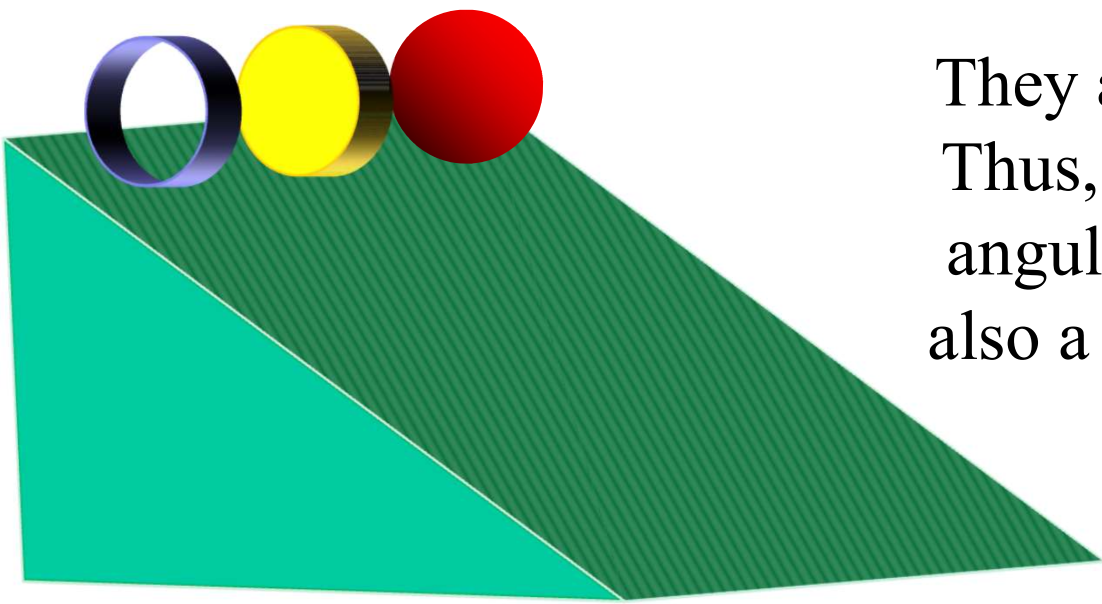
C. Sphere

D. All same

I= MR^2 1/2 MR^2 2/5 MR^2

Hoop Disk Sphere

$$\Sigma\tau = I\alpha$$

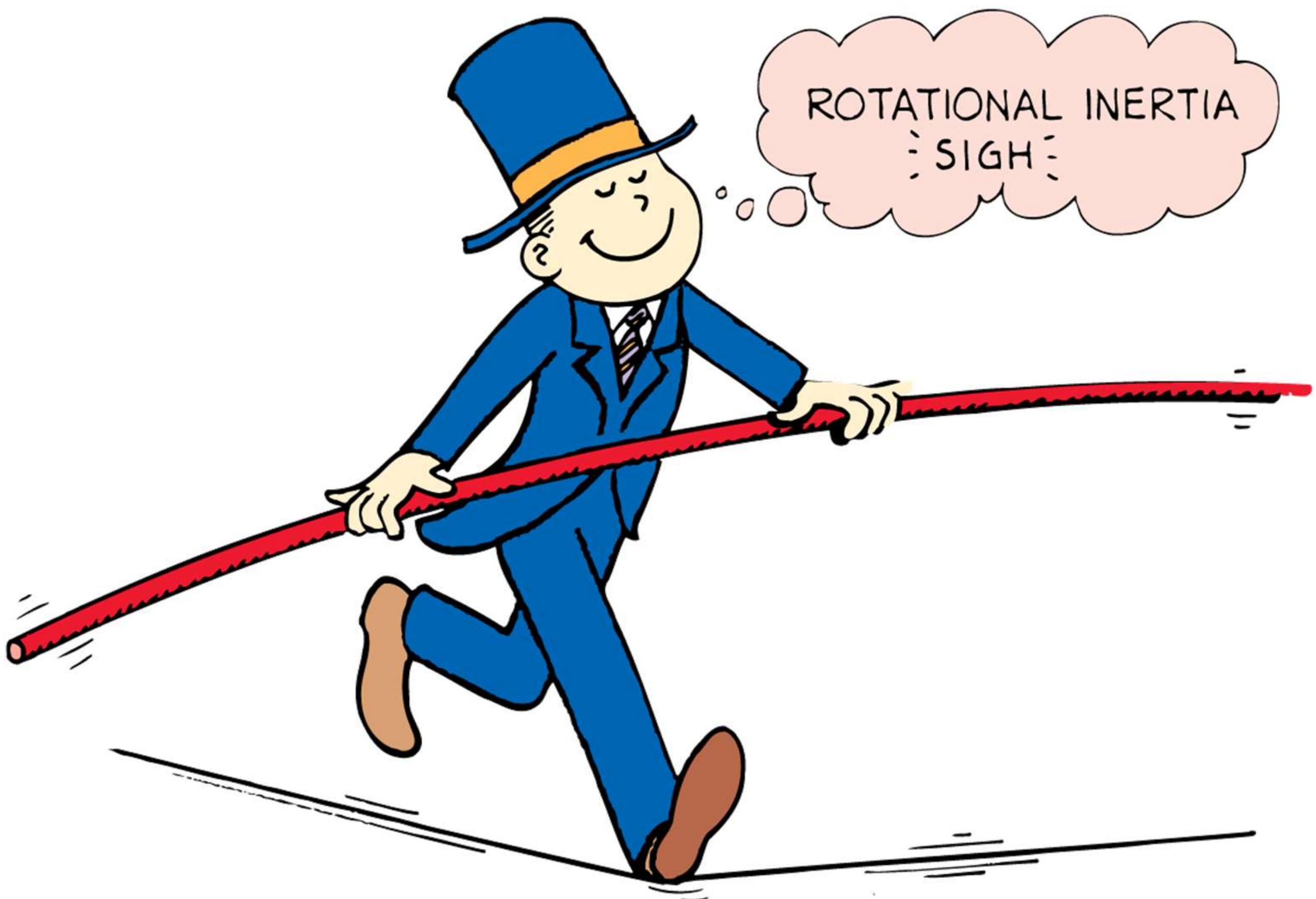


They all have the same torque. Thus, smaller I means a larger angular acceleration, which is also a larger linear acceleration

Q110

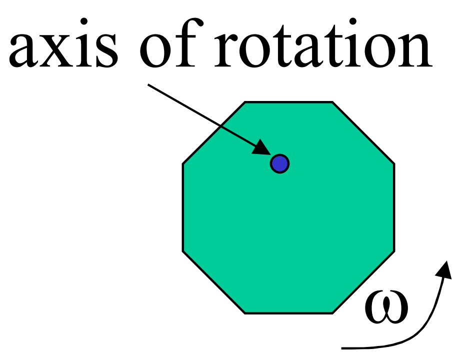
Rotational Inertia

By holding a long pole, the tightrope walker increases his rotational inertia, making it easier for him to balance (not turning).



Rotational Kinetic Energy

- Energy of rotation of a solid body



Kinetic energy is sum of kinetic energies of all particles making up the body:

$$KE = \sum \left(\tfrac{1}{2} m v^2 \right)$$

$$v = r\omega$$

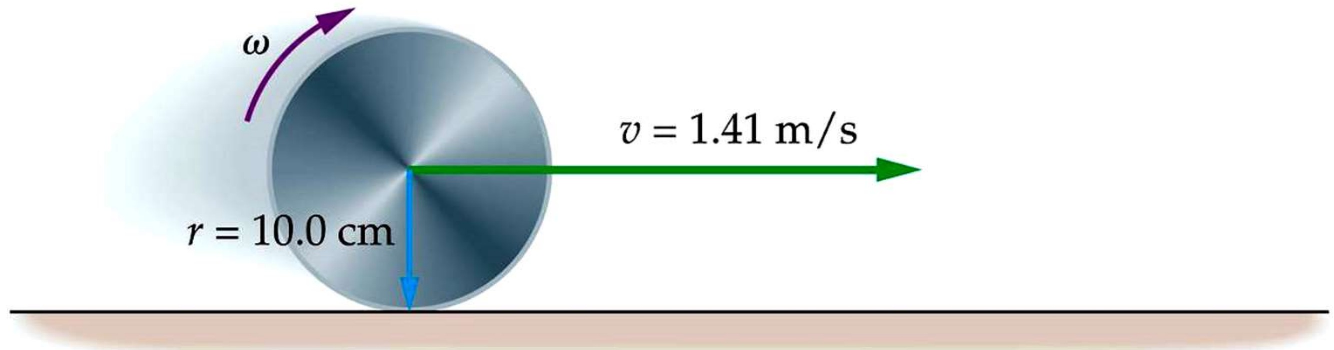
$$KE = \sum \left(\tfrac{1}{2} m r^2 \omega^2 \right) = \tfrac{1}{2} \sum \left(m r^2 \right) \omega^2$$

$$I = \sum \left(m r^2 \right)$$

Rotational KE $= \tfrac{1}{2} I \omega^2$

Like a Rolling Disk

It requires extra energy to rotate things!



A 1.20 kg disk with a radius 0f 10.0 cm rolls without slipping. The linear speed of the disk is v = 1.41 m/s.

(a) Find the translational kinetic energy.

(b) Find the rotational kinetic energy.

(c) Find the total kinetic energy.

Keep in mind for conservation of energy problems.

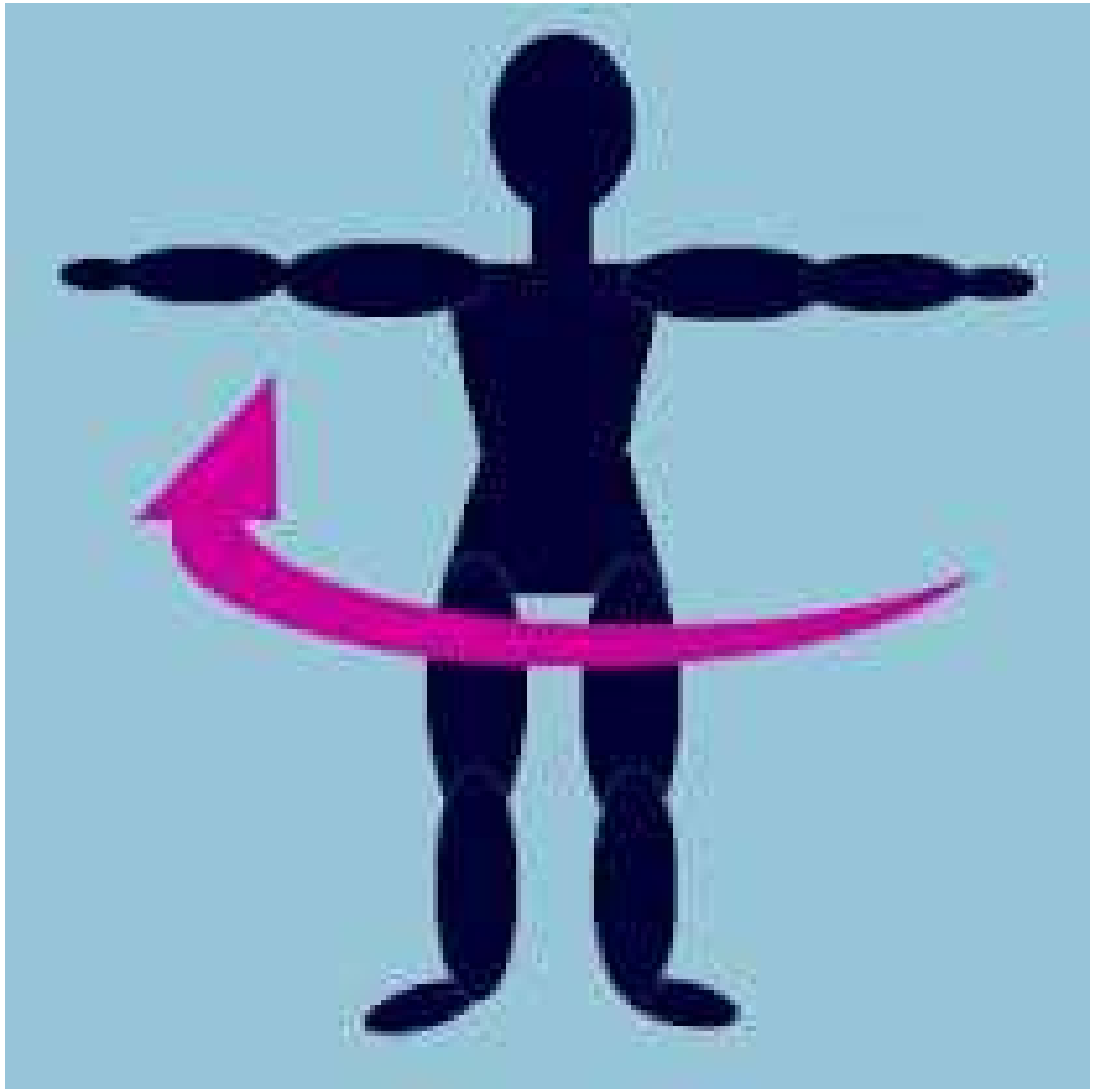
$$K_t = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.20\text{ kg})(1.41\text{ m/s})^2 = 1.19\text{ J}$$

$$K_r = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(\tfrac{1}{2}mr^2)(v/r)^2 = \tfrac{1}{4}(1.20\text{ kg})(1.41\text{ m/s})^2 = 0.595\text{ J}$$

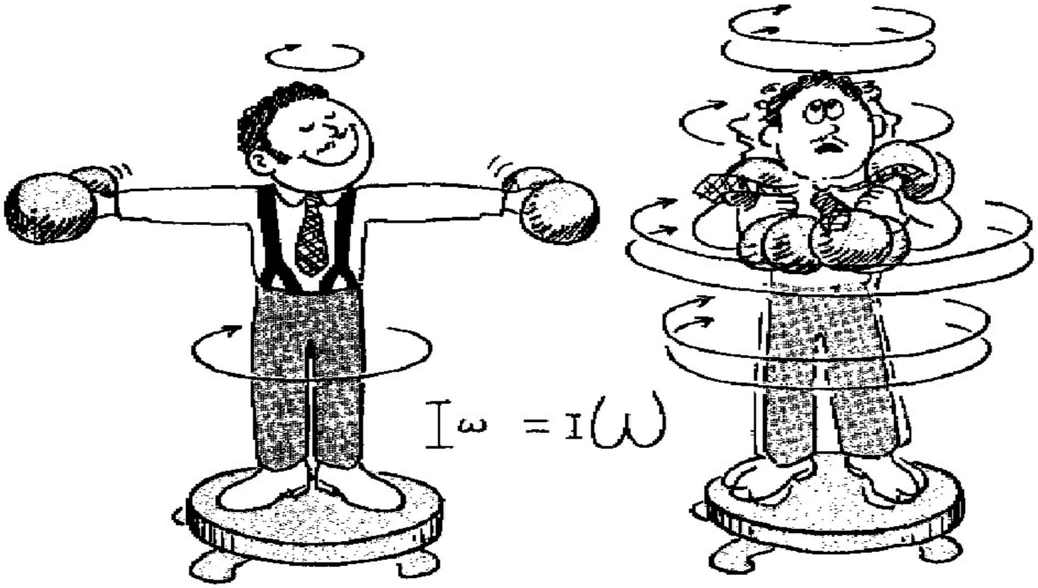
Note that the radius cancels out!

$$K_t + K_r = (1.19\text{ J}) + (0.595\text{ J}) = 1.79\text{ J}$$

Stand Up and Space Yourselves Out



What does it feel like is happening?



Conservation of Angular Momentum

Angular momentum $L = I\omega$

Spinning skater spins slowly with arms out, faster with arms in, faster yet with arms up!

Reducing r makes ω greater to have same angular momentum

$I = \Sigma MR^2$

Conservation of (Angular) Momentum

$$\vec{F}_{net} = \sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t} = 0$$

If no unbalanced torque acts on a rotating system, the angular momentum of that system is constant

$$\tau_{net,system} = \sum \tau = \frac{\Delta L}{\Delta t} = 0$$

Angular momentum:

Makes the World Go Round (pun intended)

Relations

Linear Motion	Rotational Motion
Mass m	Moment of Inertia I
Linear velocity **v**	Angular velocity ω
Translational KE $\frac{1}{2}mv^2$	Rotational KE $\frac{1}{2}I\omega^2$
Linear momentum $\mathbf{p} = m\mathbf{v}$	Angular momentum $L = I\omega$
$\vec{F}_{net} = \sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$	$\tau_{net} = \sum \tau = \frac{\Delta L}{\Delta t}$

(F=ma)
when m constant

Note: if I is constant, $\frac{\Delta L}{\Delta t} = \frac{I\omega - I\omega_o}{\Delta t} = \frac{I\Delta\omega}{\Delta t} = I\alpha$

I is likely to be constant than mass

A Windmill

In a light wind, a windmill experiences a constant torque of 255 N m.

If the windmill is initially at rest, what is its angular momentum after 2.00 s?

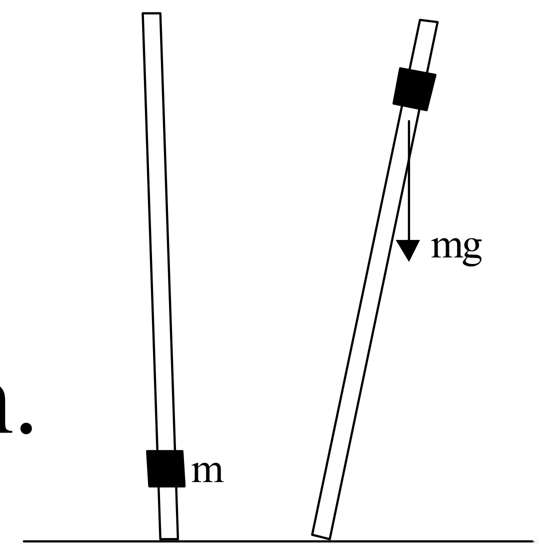
$$\tau = \frac{\Delta L}{\Delta t}$$

$$\Delta L = \tau \, \Delta t = (255 \text{ N})(2.00 \text{ s}) = 510 \text{ kg m}^2/\text{s}$$

Notice that you do not need to know the moment of inertia of the windmill to do this calculation.

A mass m is attached to a long, massless rod. The mass is close to one end of the rod. Is it easier to balance the rod on end with the mass near the top or near the bottom?

Hint: Small α means sluggish behavior and $\alpha = \frac{\tau}{I}$



A: easier with mass near top.

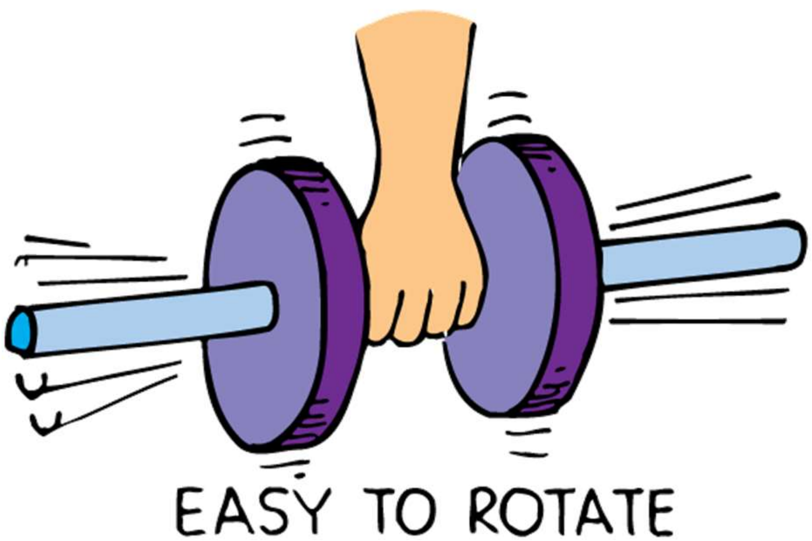
B: easier with mass near bottom.

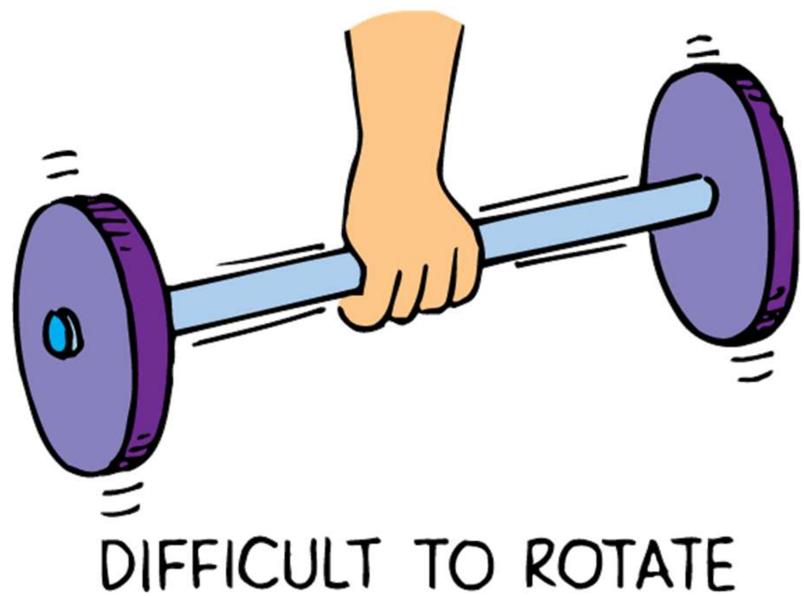
C: No difference.

Q111

Rotational Inertia

Rotational inertia depends on the distance of mass from the axis of rotation.





Race of the Geometrical Shapes

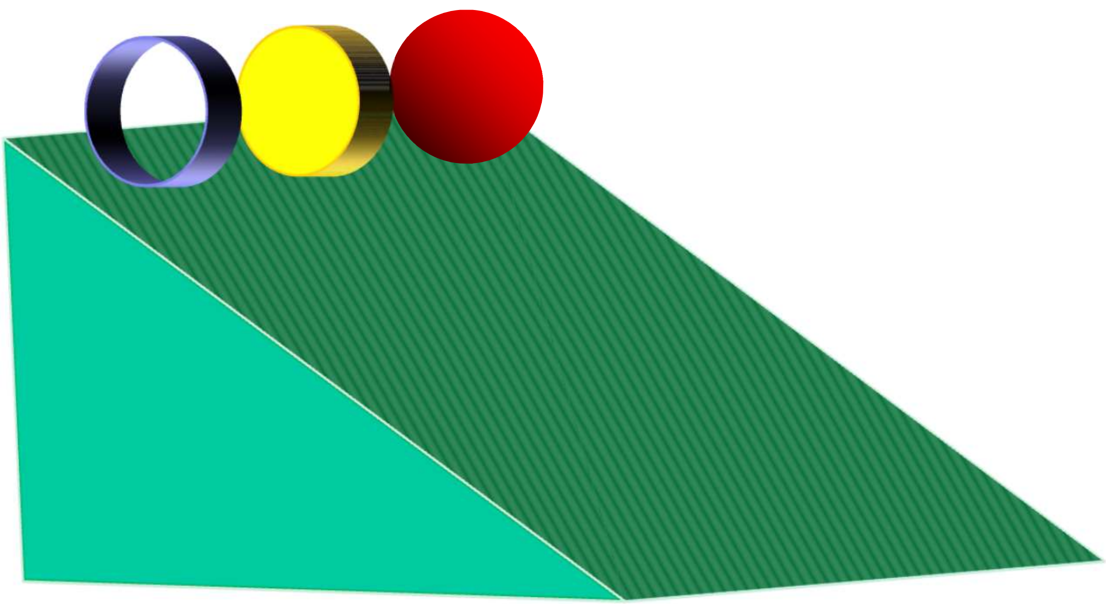
All shapes below have the same mass and same radius

Let's find the final velocities of each of these shapes.

I= MR^2 1/2 MR^2 2/5 MR^2

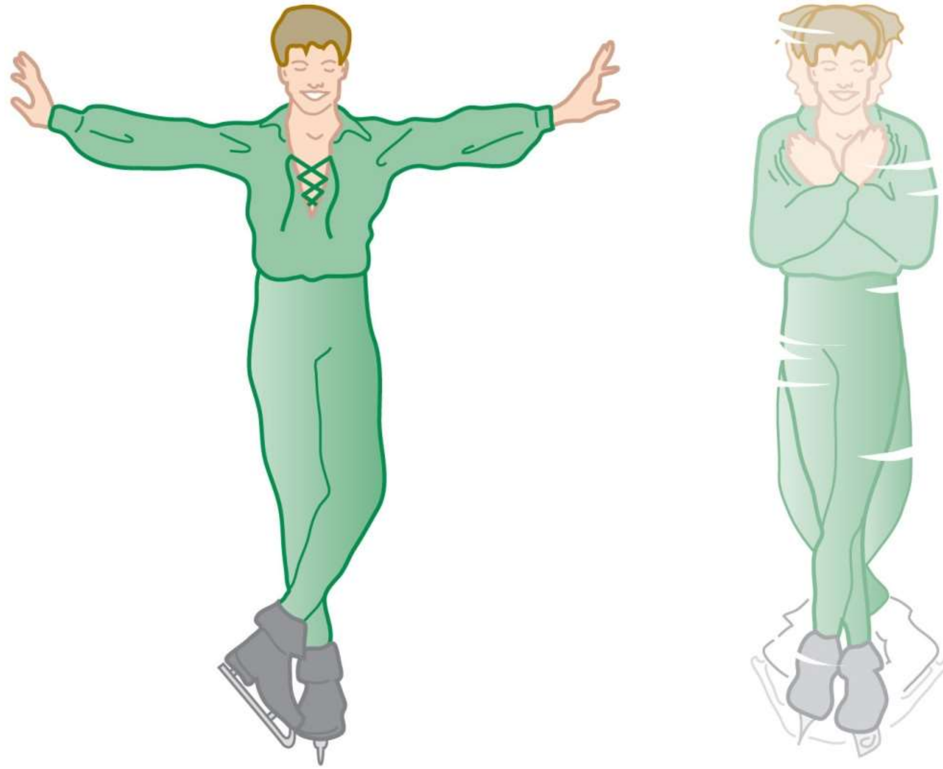
Hoop Disk Sphere

How would we do that?



How did we do this with blocks?

A spinning figure skater pulls his arms in as he rotates on the ice. As he pulls his arms in, what happens to his angular momentum L and kinetic energy K?



A. L and K both increase.

B. L stays the same; K increases.

C. L increases; K stays the same.

D. L and K both stay the same.

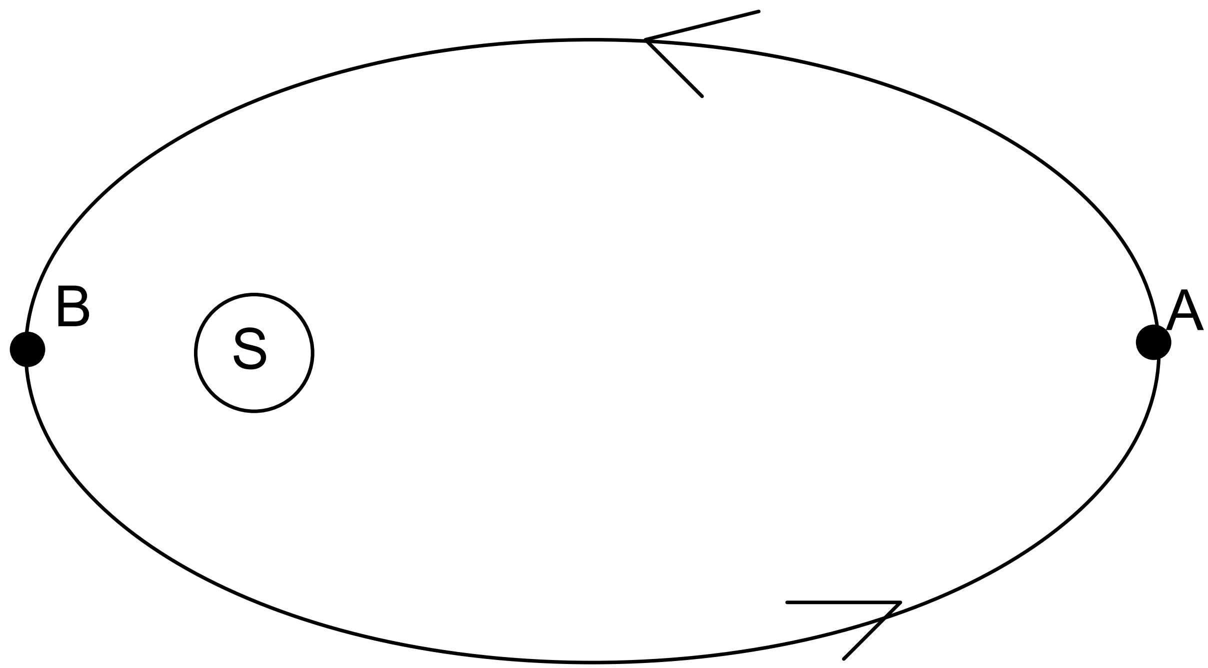
Q112

How does the magnitudes of the angular momentum of the planet L_{planet} (with the origin at the Sun) at positions A and B compare?

A: $L_A = L_B$

B: $L_A > L_B$

C: $L_A < L_B$



Rotational KE $\frac{1}{2} I\omega^2$

$I = \Sigma MR^2$

$$\frac{\Delta L}{\Delta t} = \frac{I\omega - I\omega_o}{\Delta t} = \frac{I\Delta\omega}{\Delta t} = I\alpha$$

Angular momentum $L = I\omega$

$$\tau_{net} = \sum \tau = \frac{\Delta L}{\Delta t}$$

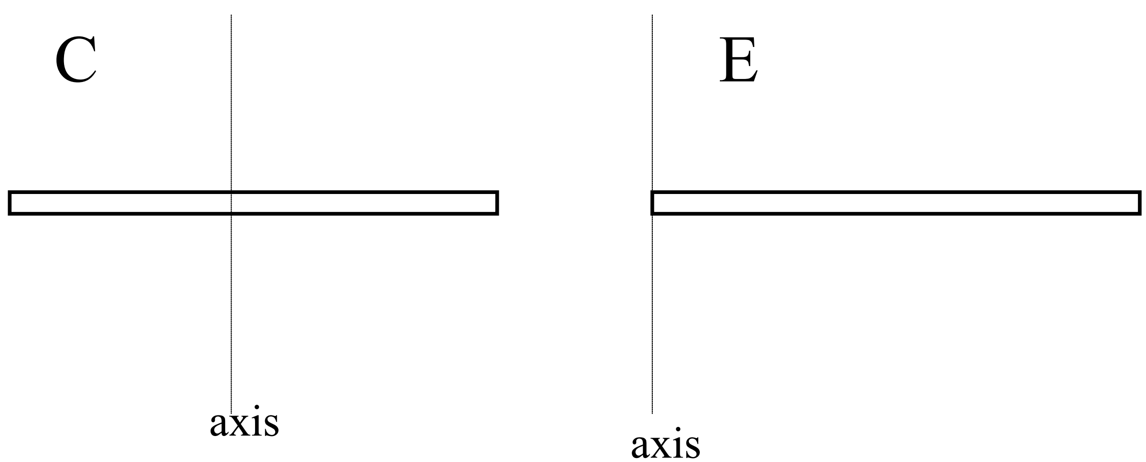
Q113

Consider a rod of uniform density with an axis of rotation through its center and an identical rod with the axis of rotation through one end. Which has the larger moment of inertia (more difficult to rotate)?

A: $I_C > I_E$

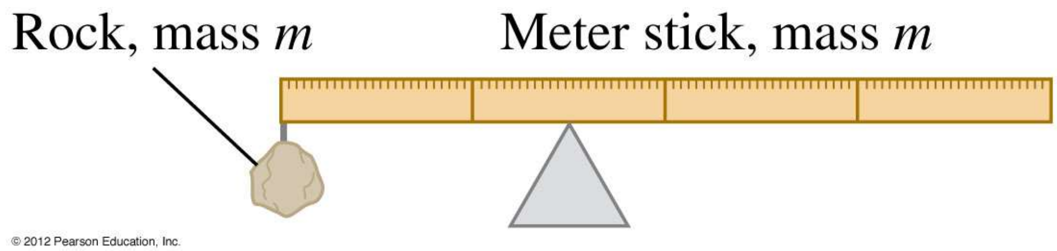
B : $I_C < I_E$

C : $I_C = I_E$



Q114

A rock is attached to the left end of a uniform meter stick that has the same mass as the rock. How far from the left end of the stick should the triangular object be placed so that the combination of meter stick and rock is in balance?



Current picture of location of the support triangle not necessarily shown correctly

A. less than 0.25 m

B. 0.25 m

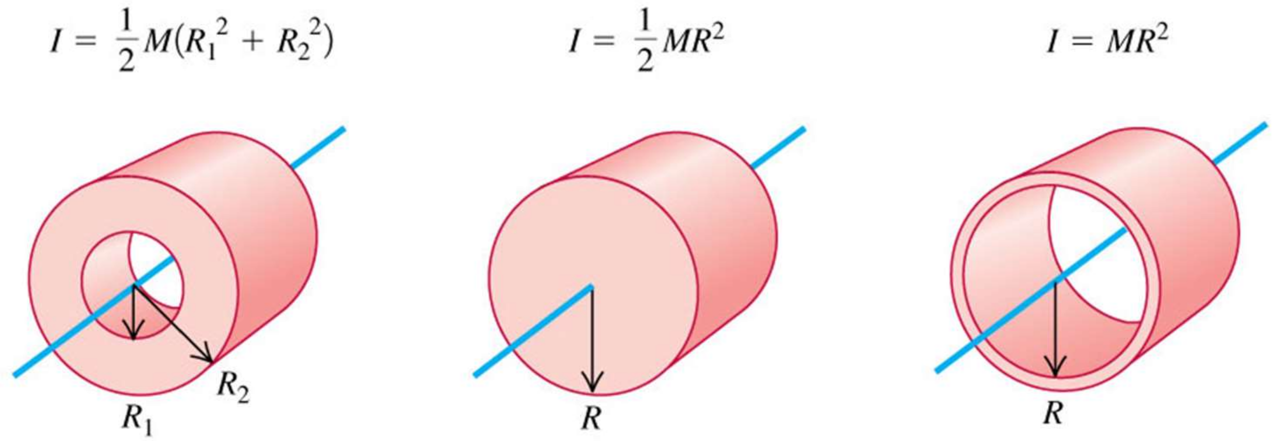
C. between 0.25 m and 0.50 m

D. 0.50 m

E. more than 0.50 m

Q115

The three objects shown here all have the same mass M and radius R. Each object is rotating about its axis of symmetry (shown in blue). All three objects have the *same* rotational kinetic energy. Which one is rotating *fastest*?



A. thin-walled hollow cylinder

B. solid sphere

C. thin-walled hollow sphere

D. two or more of these are tied for fastest

Q116

Example: A Stellar Performance

A star of radius R_i = 2.3 x 10^8 m rotates initially with an angular speed of ω_i = 2.4 x 10^{-6} rad/s.

If the star collapses to a neutron star of radius R_f = 20.0 km, what will be its final angular speed ω_f?

$$L_i = L_f \quad \Rightarrow \quad I_i\omega_i = I_f\omega_f$$

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \frac{\frac{2}{5}MR_i^2}{\frac{2}{5}MR_f^2}\omega_i = \left(\frac{R_i}{R_f}\right)^2\omega_i$$

$$= \left[\frac{(2.3\times10^8 \text{ m})}{(2.0\times10^4 \text{ m})}\right]^2(2.4\times10^{-6} \text{ rad/s}) = 320 \text{ rad/s}$$

$$= 3056 \text{ rpm}$$